

## Session 1, Paper 2

# Roller bearings lifetime under different load conditions by a semi-Markovian approach

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### Abstract

The key to a successful intelligent maintenance consists in replacing worn equipment before a failure occurs. In this context a predictive maintenance policy can be implemented if a degradation indicator is available. This paper focuses on mechanical equipment with degradation over time under various load conditions. More specifically, we will study the reliability of roller bearings under different load conditions. For a given state it is supposed that the failure rate is constant and proportional to the load. A semi-Markovian approach will be introduced in order to take into account failure rates due to the different operational states. Moreover this approach enables to take into account any holding sojourn time distribution.

**Keywords** – degradation-based reliability, semi-Markov, phase type distribution, bearing lifetime

### I. Introduction

Maintenance of industrial equipment is a lever to increase the efficiency and the productivity of industries, by minimizing the costs and by maximizing the availability.

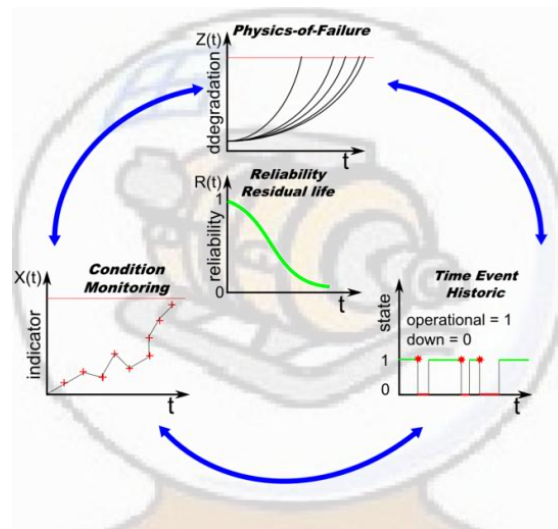


Figure 1. Three approaches to estimate reliability and mean residual life.

In order to reach the optimum, one has to know the reliability and the mean residual lifetime (*MRL*) of equipment. This information may be obtained thanks to one of the three approaches as shown in figure 1. The first one (*Time Event Historic*) is based on the record of the states of the system over time. The second approach (*Condition Monitoring*) supposes that the evolution of the degradation can be tracked by recording a health indicator on which a threshold may be fixed. The last one (*Physics-of-Failure*) supposes that the degradation process is known *a priori*, which leads to an analytical expression for the evolution of the degradation. In this paper we focus on the last approach for the design of roller bearing under different load.

## II. State of art

### II.1 Degradation based reliability background

We focus our work on the reliability. If  $T_c$  is a stochastic failure time, the reliability  $R(t)$  is:

$$R(t) = \mathbb{P}(T_c > t)$$

The degradation based approach considers that the evolution of a degradation process  $Z(t)$  may be tracked over time or is known *a priori*. Linking the degradation evolution to the reliability is achieved by considering a critical threshold  $z$  that corresponds to a failure level. The reliability is then the probability for the degradation process  $Z(t)$  to stay under that threshold:

$$R(t) = \mathbb{P}(z > Z(t))$$

At the time of failure, the threshold  $z$  is reached, the reliability is obtained by:

$$R(t) = \mathbb{P}(Z^{-1}(z) > t)$$

### II.2 Markovian approach

The Markov theory is a common approach to consider transition state problems for stochastic processes. Among the recent models, we find:

- Markov Chain which has a discrete countable state-space. In this model transitions occur at each time step increment. It is called Discrete Time Markov Process (*DTMP*).
- Continuous Time Markov Process (*CTMP*) for which the difference is that rather than transitioning to a new state at each time step, the system will remain in the current state for some random amount of time. When the distribution of sojourn time is exponential this model is also called Homogeneous Markov Process e.g. the transition rates are constant over time
- Semi-Markov Process (*SMP*) or Non Homogeneous Markov Process (*NHMP*) for which the embedded jump chain is a Markov Chain and

where the sojourn time in states are random variables with any distribution and whose distribution function may depend on the two states between which the move is made.

- Hidden Markov Model (*HMM*) is a statistical Markov model in which the system has unobservable states but the output, which depends on the state, is observable.

Several authors used Markov models to assess the reliability and maintainability of equipment. Sadek & Limnios [1] proposed a nonparametric estimation of reliability and survival function for continuous-time finite Markov processes. Guo & Yang [2] proposed an automatic creation of Markov models for reliability assessment of safety instrumented systems. Zhao & al. [3] developed condition-based inspection/replacement policies for non-monotone deteriorating systems with time homogeneous Markov chain environmental covariates. Bloch-Mercier [4] studied a preventive maintenance policy with sequential checking procedure for a Markov deteriorating system.

Implementing a degradation-based reliability with a Markovian approach may be achieved by two ways. The first one consists in considering a Markov chain with some functional states and an absorbing state that corresponds to the failure level [5]; the reliability is then the probability over time for the Markov chain to reach that absorbing state. The second one refers to a multistate degradation that consists in choosing several wear/failure rates according to the load case of the different states [6] ; thus each state contributes to the degradation according to its wear rate. A threshold is then fixed and we simulate the Markov chain to calculate the cumulated damage until it reaches the given threshold.

On the semi-Markov applications, Kharoufeh proposed a semi-Markov model for degradation-based reliability [6]; the estimation of sojourn time is achieved by using phase type distributions. Solo [7] presented a resume of this methodology and used it to represent the crack growth of a turbine blade under different load cases. Ouhbi [8] proposed a non-parametric reliability estimation of semi-Markov processes. Lefebvre [9] studied a semi-Markov process with an inverse Gaussian distribution as sojourn time.

### III. Methodology

#### III.1 Hypotheses and notations

We consider an item of equipment that works under various load cases, which are represented by different and distinct states. For each state, a constant degradation rate  $r(i)$  is fixed defining the degradation rate vector  $\mathbf{R} = [r(1), r(2), \dots, r(n)]$  ; those rates can correspond to a *Physic-of-Failure* model of the degradation (e.g. a crack growth function of the applied stress) or to a failure rate if we know the mean time to failure of the component for a given load case. Then a degradation threshold is fixed which can correspond to a specific value of the degradation process (e.g. a critical crack length) or to the accumulated damage that reaches 1 according to fatigue law.

The state space  $E$  is a finite vector of  $n$  distinct states. The process starts from a given state defined by the user and then visits a state  $i \in E$  and spends a random amount of time before a new transition to another state  $j \neq i$ . The transition rates between states are defined in a transition probability matrix  $\mathbf{P}$ . This matrix can be estimated from field data or may be defined in the component design phase. The time spent in state  $i$  is a stochastic value that depends on a distribution or recorded time from field data.

### III.2 Estimating the Transition Matrix from field data

We assume that the load cases are known over some time interval  $[0, t]$  and that the equipment has not failed within this interval. When a transition occurs, the transition time, the current state and the destination state are recorded. Let  $T(i, j)$  be the true rate of transition from state  $i$  to state  $j$ ,  $i \neq j$ . Let  $X_t(i, j)$  be the number of transitions from  $i$  to  $j$  and  $V_t(i)$  the total number of visits to state  $i$  over the time interval. If the time interval is long enough, we can show that:

$$T(i, j) \approx \hat{T}_t(i, j) = \frac{X_t(i, j)}{V_t(i)}, \quad j \neq i$$

$$\hat{T}_t(i, i) = - \sum_{j \neq i} \hat{T}_t(i, j)$$

If  $\hat{\mathbf{P}}$  is the estimated transition matrix, the estimated transition rates  $\hat{p}_{i,j}$  are:

$$\hat{p}_{i,j} = \begin{cases} \frac{\hat{T}_t(i, j)}{-\hat{T}_t(i, i)} & \text{if } j \neq i \\ 0 & \text{if } j = i \end{cases}$$

### III.3 Phase-Type distribution

When the sojourn times are not exponential, a common technique is to use phase-type approximation to estimate an underlying Markov process from the observed time data. Solo and Kharoufeh presented a good summarize of this methodology [6], [7]. We briefly report the main idea in this paper.

A phase-type distribution is a probability distribution that results from a system of one or more inter-related Poisson processes occurring in a stochastic sequence like a Markov chain [10]. The distribution can be represented by a random variable describing the time until absorption of a Markov process with one absorbing state. Each of the states of the Markov process represents one of the phases.

Consider a continuous-time Markov process with  $m+1$  states, where  $m \geq 1$ . The states  $1, \dots, m$  are transient states and state  $m+1$  is an absorbing state. The continuous phase-type distribution is the distribution of time from the process starting until it reaches the absorbing state. This process can be written in the form of a transition rate matrix  $\mathbf{Q}$  such as

$$Q = \begin{bmatrix} S & S^0 \\ O & 0 \end{bmatrix}$$

Where  $S$  is a  $m \times m$  matrix,  $O$  is a null vector  $1 \times m$ ,  $S^0$  a column vector  $m \times 1$  so as  $S.e + S^0 = O$ ,  $e$  being a column vector of 1. The vector  $\alpha_0 = (\alpha, \alpha_{m+1})$  is the initial distribution vector of  $Q$ , i.e.,  $\alpha$  is a  $1 \times m$  vector and  $\alpha_{m+1}$  is the probability that the Markov process begins in the absorbing state  $m+1$ . The cumulative function of a random variable  $Y$  following a phase type distribution is:

$$F(y) = P(Y \leq y) = 1 - \alpha \exp(Sy) e$$

#### III.4 Specific Phase Type distribution

In order to choose a specific Phase Type distribution for each state, Kharoufeh [6] proposed the following decision frame (table 1) depending on the square coefficient of variation obtained from the sojourn times.

**Table 1 Phase Type approximation selection criteria**

Range of $c^2$	Phase Type approximation
$0 < c^2 < 0,5$	2-moment, $k$ -Phase generalized Erlang
$0,5 \leq c^2 \leq 1$	2-moment, 2-Phase Coxian
$c^2 > 1$	3-moment, 2-Phase Coxian

For a  $k$ -Phase Type distribution, it requires to calculate the  $k^{th}$  first moment and variance from the sojourn times  $D(i)$  observed for each state  $i$ .

#### III.5 Implementing Phase Types distributions in main Markov transition matrix

The cardinality of the initial state space  $E$  is  $N$  due the  $n$  possible states of the initial Markov chain (see III.1). As we introduce Phase Type distribution, the cardinality  $\hat{N}$  of the new state space  $\hat{E}$  has to be calculated according to the fact that we replace a state  $i$  by an absorbing Markov Chain with  $k_i + 1$  states ( $k_i$  being the number of phases). Thus the new state space has cardinality

$$|\hat{E}| = N + \sum_{i=1}^n k_i$$

The new transition matrix  $\hat{P}$  has the form:

$$\hat{P} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$

with

$$A_{ii} = \begin{bmatrix} S_i & S_i^0 \\ \mathbf{0} & c\hat{T}_t(i, i) \end{bmatrix}$$

And for  $i \neq j$

$$A_{ij} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ c\hat{T}_t(i, j) & \mathbf{0} \end{bmatrix}$$

$c$  is a very large positive number that ensure that the process will instantaneously transit to the next state chosen by  $\hat{P}$  when the sojourn in state  $i$  is complete.  $S_i$  is as  $k_i \times k_i$  matrix while  $S_i^0$  is a column vector of size  $k_i$ . As we have increase the size of the Markov transition matrix, we also have to change the size of degradation rate vector as follow

$$\hat{R} = \begin{bmatrix} \Delta(r(1)) & 0 & \dots & 0 \\ 0 & \Delta(r(2)) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Delta(r(n)) \end{bmatrix}$$

With  $\Delta(r(i))$  being a diagonal matrix  $k_i+1 \times k_i+1$  whose diagonal entries are all  $r(i)$ . The initial vector  $\alpha$  also have to be changed by

$$\hat{\alpha} = (1, 0, 0, \dots, 0)$$

Such that the length of  $\hat{\alpha}$  is equal to the cardinality of the new state space  $\hat{E}$

### III.6 Calculation of the lifetime distribution

The initial semi-Markov process with state space  $E$  and transition matrix  $P$  has been replaced by a continuous time Markov chain with state space  $\hat{E}$  and transition matrix  $\hat{P}$ . The unconditional lifetime distribution of a continuous time Markov chain  $X(t)$  and a monotonous degradation process  $Z(t)$  with a failure threshold  $z$  is:

$$F(z, t) = \mathbb{P}(T_z \leq t)$$

We define

$$V(z, t) = [V_{i,j}(z, t)]$$

a square matrix with the cardinality of the new state space  $|\hat{E}|$  and

$$V_{i,j}(z, t) = \mathbb{P}(Z(t) \leq z, X(t) = j | X(0) = i)$$

It represents the joint probability that at time  $t$ , the degradation of the system has not exceeded the given threshold  $z$  and that the process is in state  $j$  given that it was initially in state  $i$ . When the degradation rates are nonnegative, the above joint probability becomes [7]:

$$\hat{V}^*(u, s) = [u\hat{R} + s\mathbf{I} - \hat{P}]^{-1}$$

With  $Re(u) > 0, Re(s) > 0, \mathbf{I}$  the identity matrix. The failure time distribution is given by

$$\hat{F}^*(u, s) = \frac{1}{s} - \hat{\alpha} \hat{V}^*(u, s) \hat{e}$$

$\hat{e}$  is a column of 1 with cardinality of the new state space  $\hat{E}$ . Kharoufeh and Sipe [11] proposed a one-dimensional Laplace transform with respect to  $t$  so that the failure function for a given threshold  $z$  is

$$\hat{F}_z(s) = \hat{\alpha} \exp(\hat{R}^{-1}(\hat{P} - s\mathbf{I})z) \mathbf{e}$$

Solo [7] proposed an algorithm for the inverse Laplace transform in order to obtain the reliability function in the time domain.

#### IV. Bearing lifetime

The calculation method for dynamically stressed rolling bearings is based on material fatigue as the cause of failure. According to the DIN ISO 281, the life formula is [12]:

$$L_{10}[\text{Mcycles}] = L[\text{Mcycles}] = \left(\frac{C}{P}\right)^m$$

With  $L_{10}$  being the nominal rating life in millions of revolutions which is reached or exceeded by at least 90% of a large set of identical bearing.  $C$  is the dynamic load rating defined by the constructor,  $P$  the equivalent load rating induced by the application,  $m$  the life exponent that is worth 3 for ball bearings and 10/3 for roller bearings and needle bearing. The nominal rating life is valid for bearings made of conventional roller bearing steel and usual operating conditions (good lubrication, no extreme temperatures,...). When the conditions differ from the usual ones, corrective factors are used to obtain the adjusted rating life. The attainable life becomes  $L_{na} = a_1 a_{23} L$  where  $a_1$  is the reliability factor; the DIN ISO 281 specifies the values for different reliability level. In this paper, we consider a reliability factor of 99% which corresponds to  $a_1 = 0,21$ .  $a_{23}$  is the life adjustment factor that takes into account lubricating conditions. In this paper we suppose that the bearings are working under good conditions so that we can fix the value of  $a_{23} = 1$ . If the quantities influencing the bearing life (e.g. load, operating conditions ...) are variable, the attainable life under the different constant conditions has to be determined for every operating time  $q_i$  [%]. The attainable life under  $n$  different operating conditions is then obtained by using the formula [12]

$$L_{na} = \frac{100}{\frac{q_1}{L_{na1}} + \frac{q_2}{L_{na2}} + \dots + \frac{q_n}{L_{nan}}}$$

For each operating condition, the inverse of the attainable life may be considered as a failure rate which means that each revolution contributes to



the cumulative damage according to its own failure rate. That being said, the degradation rate vector for each state defined in III.1 is :

$$\mathbf{R} = \left[ r(1) = \frac{1}{L_{na1}}, r(2) = \frac{1}{L_{na2}}, \dots, r(n) = \frac{1}{L_{nan}} \right]$$

## V. Illustrative examples

The application concerns the bearing of a CNC lathe spindle where we represent a continuous density function of relative forces by a finite number of states. A discussion is proposed to study the sensibility of the number of states.

The case concerns the study of bearings in a machine spindle. These machines function under different load conditions according to the cutting parameters. Some authors have investigated the reliability of machines. Yazhou & al. [13] presented a reliability approach to machine tool bearings by considering a Weibull distribution with dynamic load rating to represent the times of failure. Wang & al. [14] made a survey over some CNC lathe to determine the causes of machine failures ; 69% of broken components were electronics, 26% were mechanical components and 5% were hydraulics/pneumatics components.

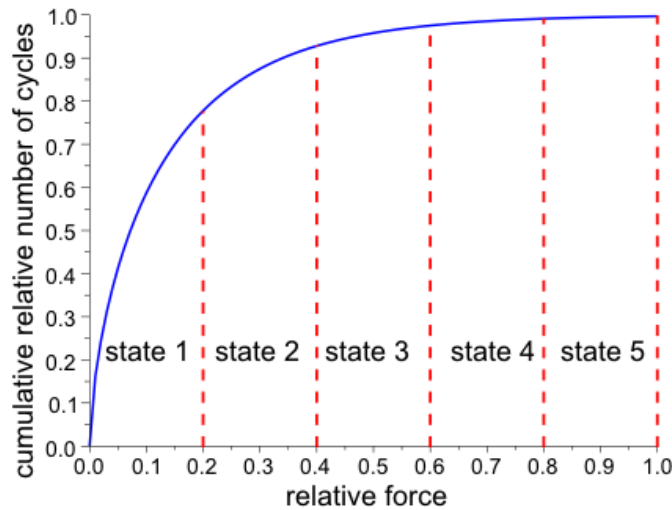
We suppose that the spindle has been designed for the maximum cutting forces. The table 4 summarizes the values.

**Table 4 Maximum cutting forces**

$F_r$ [N]	$F_t$ [N]	Torque $M$ [Nm]	Spindle speed $M$ [rev/min]	Power [W]	Bearing radial reaction $P_{max}$ [N]
1591	2651	83,5	1137	9943	6136

Obviously the spindle won't work at maximum power all the time. To take into account different load cases, we introduce the cumulative density function (*cdf*) of relative force spectrum proposed by Wang [15]. From a database of 112 medium-sized CNC machine tools over 3 years, Wang modeled the distribution of relative force with a gamma distribution with shape parameter 0,629 and scale parameter 4,756. We then consider a finite number of states in the *cdf* curve. The figure 4 shows the gamma cumulative density function of relative forces with 5 states subdivision.





**Figure 4. gamma cumulative density function of relative force according to Wang and 5 states subdivision**

From the *cdf* probabilities we construct the transition matrix. As an example we consider the case with  $n=5$  states, the transition probability from state  $i$  to state  $j$  is given by:

$$p(i, j) = \frac{cdf(j+1) - cdf(j)}{1 - (cdf(i+1) - cdf(i))}$$

With  $cdf(1) = 0$  and  $cdf(n+1) = 1$ . The transition matrix for a 5 states representation of the *cdf* force is

$$P = \begin{bmatrix} 0 & 0,6755 & 0,2127 & 0,0722 & 0,0396 \\ 0,9152 & 0 & 0,0556 & 0,0189 & 0,0104 \\ 0,8164 & 0,1575 & 0 & 0,0168 & 0,0092 \\ 0,7905 & 0,1525 & 0,0480 & 0 & 0,0089 \\ 0,7847 & 0,1514 & 0,0477 & 0,0162 & 0 \end{bmatrix}$$

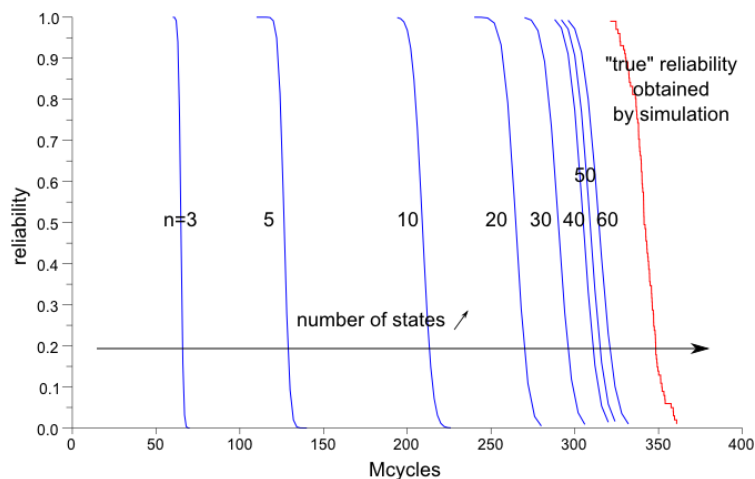
Degradation rates are calculated from the relative force spectrum. We choose the median value of each state  $i$  such as  $P(i) = P_{max}((i/n + (i-1)/n)/2)$ . We consider a bearing with a dynamic load capacity of 19,30kN. The table 5 presents the degradation rates as well as estimated lifetime for each state ( $a_1 = 0,21$ ,  $a_{23} = 1$  and  $m = 3$ ).

**Table 5 Degradation rates for a 5 states subdivision**

State	$P[N]$	$L_{99}[Mc]$	$r[1/Mc]$
1	613,6	6534,84	0,00015
2	1840,8	242,03	0,00413
3	3068	52,28	0,01913
4	4295,2	19,05	0,05249
5	5522,4	8,96	0,11156

Finally we consider a Weibull distribution with scale 10000 cycles and shape 1.5 to simulate sojourn times for each state. We proceed on the reliability calculation for several number of states subdivision. We also calculate the non-parametric reliability obtained by simulation based on the “true” gamma distribution of the relative force. The figure 5 shows the reliability curves for different number of states. We see that the higher the number of states the more accurate the reliability is. However there is still a gap between the true reliability obtained by the simulation of the gamma distribution (red staircase).

A possible explanation for that phenomenon is that the force hasn't a linear effect on the bearing lifetime but according to a 3<sup>rd</sup> order power law. Moreover, the lesser the state space the shorter the lifetime will be as we calculate the lifetime for the upper value of each state so that we may obtain a lower bound for the reliability.



**Figure 5. semi-Markov reliabilities as a function of the number of states. The red staircase reliability is the non-parametric reliability obtained by the simulation of degradation paths with the “true” gamma distribution of the relative force.**

## VI. Summary & Conclusion

In this paper, we presented a methodology for reliability calculation under different stochastic load cases with any sojourn time distribution. The literature review presented the Markovian approach and more specifically detailed the semi-Markovian approach by a Phase-Type representation. Kharoufeh [6] and [7] set up a strong background for this methodology. The objective of this paper was to study the expectations of that methodology when the number of states to represent a continuous distribution function is unknown. We focused on bearing lifetime according to fatigue law. As a perspective this methodology should be tested on real field data. Items of equipment are meant to be used under various load conditions and this methodology could be used to predict the reliability during the design phase and to actualize it during exploitation phase.

## References

- [1] Sadek & Limnios, Nonparametric estimation of reliability and survival function for continuous-time finite Markov processes, *Journal of Statistical Planning and Inference* 133 p. 1-21, 2005.
- [2] Guo & Yang, Automatic creation of Markov models for reliability assessment of safety instrumented systems, *Reliability Engineering & System Safety* 93 p. 807-815, 2007.
- [3] Zhao & al., Condition-based inspection/replacement policies for non-monotone deteriorating systems with environmental covariates, *Reliability Engineering and System Safety* 95 p. 921-934, 2010.
- [4] Bloch-Mercier, A preventive maintenance policy with sequential checking procedure for a Markov deteriorating system, *European Journal of Operational Research (Stochastic and Statistics)* 147 p. 548-576, 2002.
- [5] Muhammad & al., Reliability evaluation for a multi-state system subject to imperfect repair and maintenance, *International Journal of Engineering & Technology IJET-IJENS* Vol.10 No 01, 2010
- [6] Kharoufeh & al., Semi-Markov Models for Degradation-Based Reliability, *Forthcoming in IIE Transactions*, 2009
- [7] Solo, *Phase-Type approximations for wear processes in a semi-Markov environment*, Thesis, Air Force Institute of Technology Ohio, 2004.
- [8] Ouhbi & Limnios, Non parametric reliability estimation of semi-Markov processes, *Journal of Statistical Planning and Inference* 109 p. 155-165, 2003.
- [9] Lefebvre & Perotto, A semi-Markov process with an inverse Gaussian distribution as sojourn time, *Applied Mathematical Modelling* 35 p. 4603-4610, 2011.
- [10] Drekic, *Phase-Type distribution*, Encyclopedia of Quantitative Finance, Wiley Online Library, 2010.
- [11] Kharoufeh & Sipe, Evaluating failure time probabilities for a Markovian wear process, *Forthcoming in Computers and Operations Research*, 2004.
- [12] *The Design of Rolling Bearing Mountings*, FAG Bearing Limited, Publ. No. WL 00 200/5 EC.
- [13] Yazhou & al., A reliability approach to machine tool bearings, *Reliability Engineering and System Safety* 50 p. 127-134, 1995.
- [14] Wang & al., Failure probabilistic model of CNC lathes, *Reliability Engineering and System Safety* 65 p. 307-314, 1999.
- [15] Wang & al., Load spectra of CNC machine tools, *Quality and Reliability Engineering International* 16 p. 229-234, 2000.